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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, JUNE 2022
THIRD YEAR (BATCH 2019-22)
MATHEMATICS (HONOURS)
Paper : XIV [CC14]

Date : 17/06/2022
Time : 11.00 am – 1.00 pm
Full Marks : 50

[Use a separate Answer book for **each group**]

Group – A

Answer **all** questions, maximum you score is 20 marks.

1. Write the equation $x^2 z_{xx} - y^2 z_{yy} = 0 (x \neq 0, y \neq 0)$ in equivalent canonical form. [5]
2. Solve the following wave equation of a string of finite length using the method of separation of variables :
   $$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0,\ 0 \le x \le L,\ t \ge 0$$
   subject to the initial conditions $u(x,0) = f(x)$, $u_t(x,0) = g(x)$, $0 \le x \le L$ and the boundary conditions $u(0,t) = u(L,t) = 0,\ t \ge 0$. [6]
3. Solve the following heat equation for a finite rod by the method of separation of variables:
   $$\frac{\partial T}{\partial t} - 4\frac{\partial^2 T}{\partial x^2} = 0,\ 0 \le x \le 2,\ t \ge 0$$
   subject to the boundary conditions
   $$T(0,t) = T(2,t) = 0, t \ge 0$$
   and the initial condition $T(x,0) = x,\ 0 \le x \le 2$. [5]
4. Using Laplace Transform technique solve the equation
   $$\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial t} + y,\ y(x,0) = 6e^{-3x}$$
   which is bounded for $x > 0,\ t > 0$. [4]
5. Obtain the D'Alembert's formula for the problem $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0, -\infty < x < \infty, t \ge 0$

   subject to $u(x,0) = f(x), u_t(x,0) = g(x)$. [4]

Group – B

Answer **all** the questions, maximum one can score is 30 marks.

6. Let f be an entire function such that $f(z) = 0 \forall z \in D$, where $D = \{z \in \mathbb{C} : |z-3| = 5\}$. Show that f is a constant function. [3]
7. Show that the general bilinear transformation which maps the circle $K : |z| = r$ onto the circle $K' : |w| = r'$ and Int(K) mapped into Int(K$'$) is given by

   $w = rr'e^{i\lambda}\left(\frac{z-\alpha}{\bar{\alpha}z - r^2}\right)$ where $\lambda \in \mathbb{R}$ and $|\alpha| < r$. [6]

8. Find the Taylor series for the function $\frac{1}{z}$ about z=2. Hence Show that

$$\frac{1}{z^2}=\frac{1}{4}\sum_{n=0}^{\infty}(-1)^n(n+1)\left(\frac{z-2}{2}\right)^n \text{ when } |z-2|<2.$$ [3+3]

9. Use Liouville's theorem to show that bounded harmonic functions (from $\mathbb{R}^2$ to $\mathbb{R}$) are constant. [6]

10. Evaluate $\int_C f(z)dz$ where $f(z)=\pi e^{(\pi\bar{z})}$ and C is the boundary of the square with vertices at the points 0,1,1+i and i, which is oriented in the positive sense. [7]

11. For $z=x+iy$, show that $|\sinh y|\le|\sin z|\le\cosh y$. [5]

12. Is there any analytic function whose real part is $x^2+3xy+7$? (Justify). [3]

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